**Engineering Science**
2017; 2(1): 5-13
http://www.sciencepublishinggroup.com/j/es
doi: 10.11648/j.es.20170201.12

# The Physics of Electric Current Near Cylindrical Magnetic Interface

**Gilbert A. Ibitola[1], O. Ajanaku[1], Lawrence O. Imafidon[2], W. A. Azeez[3]**

[1]Department of Physical Sciences, Ondo State University of Science and Technology, Okitipupa, Nigeria
[2]Department of Physical Sciences, Yaba College of Technology, Yaba Lagos, Nigeria
[3]Department of Physics, The Polytechnic, Ibadan, Nigeria

**Email address:**
ibitolaieee@gmail.com (G. A. Ibitola), lanrayus@gmail.com (O. Ajanaku), lawfidon@gmail.com (L. O. Imafidon), waazeez@yahoo.com (W. A. Azeez)

**To cite this article:**
Gilbert A. Ibitola, O. Ajanaku, Lawrence O. Imafidon, W. A. Azeez. The Physics of Electric Current Near Cylindrical Magnetic Interface. *Engineering Science.* Vol. 2, No. 1, 2017, pp. 5-13. doi: 10.11648/j.es.20170201.12

**Received**: November 7, 2016; **Accepted**: December 27, 2016; **Published**: February 24, 2017

**Abstract:** This paper presents the Physics of current near a cylindrical magnetic interface. In the past, several texts and papers have focused attention on current lines near rectangular structures near conducting or insulating planes and on magnetic fields around solenoids and some other types of current-carrying conductors. This paper explains what happens when a current line passes near a cylindrical magnetic interface. The phenomenon of placing a cylinder in a uniform external magnetic field is explained. The concept of real image and virtual electric currents is analyzed and discussed. The reciprocal point as image on the cylinder is also considered. Also considered and analyzed are phenomena of magnetic force on the cylinder wall and the effective dipole moment in the far-field. This research finds applications in the designs of magnetic systems and coils for electric generators.

**Keywords:** Maxwell's Equations, Dipole, Line-Current, Magnetic Field, Cylindrical Magnetic Interface, Poisson Equations, Magnetic Permeability, Magnetic Polarization, and Electric Permittivity

## 1. Introduction

In this paper, the physics of electric current passing near a cylindrical magnetic interface will be considered. Section two of this paper outlines the concept of current passing near a conducting or insulating plane. Section three considers and analyses the effect of an external uniform magnetic field on a cylinder placed in it. The effect of line-current passing near an insulating or conducting cylinder will be considered in Section four. Section five discusses the concept of real image and virtual electric currents, while Section six identifies a reciprocal point as image on the cylinder. Section seven of the paper will discuss the identified homogeneous medium and the conducting or insulating walls, while Section eight derives the formula for the magnetic force on the wall and the effective dipole moment in the far field. In Section nine, the similarities and differences between electrostatic and magnetostatic fields will be outlined, and the phenomenon of "skin effect" and its application will be mentioned. The conclusions will be presented in Section ten.

## 2. Current Near a Conducting or Insulating Plane

The Maxwell equations for the steady magnetostatic field:

$$\nabla.\vec{B} = 0; \tag{1}$$

$$\nabla \times \vec{H} = \frac{\vec{J}}{c} \tag{2}$$

State that (i) the magnetic induction $\vec{B}$ is divergence free, since it is a solenoidal field; (ii) the curl of the magnetic field $\vec{H}$ equals the electric current density j divided by the speed of light in vacuum c. The magnetic field and induction can be non-parallel in a crystal; in an isotropic medium they must be parallel, specifying the magnetic permeability:

$$\vec{B} = \vec{H}\mu; \tag{3}$$

$$\overrightarrow{Q_m} = \vec{B} - \vec{H} = (\mu - 1)\vec{H} = \left(1 - \frac{1}{\mu}\right)\vec{B} = \chi_m\vec{H}; \tag{4}$$

$$\chi_m = \mu - 1 \quad (5)$$

Where: $\overrightarrow{Q_m} = magnetic\ polarisation;$
$\chi_m = magnetic\ susceptibility.$

The magnetic induction and field have the same direction in a linear, isotropic material, and thus the magnetic permeability is positive:

$$\mu \geq 0;\ \mu_o = 1, \quad (6)$$

The magnetic permeability is unity in a vacuum, and when the magnetic field and induction coincide, the magnetic susceptibility and polarization vanish. The magnetic energy $E_m$ is given by:

$$E_m = \frac{1}{2}\left(\vec{B}.\vec{H}\right) = \mu H^2 = \frac{B^2}{\mu} = \frac{\mu\vec{H}.\vec{Q}_m}{(\mu-1)} = \frac{\mu Q_m^2}{(\mu-1)^2} \quad (7)$$

Now, considering an electric current J at a distance "a" from a wall, the conjugate magnetic field is given, for identical image, by:

$$H_{\pm}^{*}(z) = \frac{iJ}{2\pi c}\left[(z-ia)^{-1} \pm (z+ia)^{-1}\right], \quad (8)$$

which simplifies to:

$$H_{+}^{*}(z) = \frac{iJ}{\pi c}\frac{Z}{(z^2+a^2)}, \quad (9)$$

$$H_{-}^{*}(z) = \frac{-J}{\pi c}\frac{a}{(z^2+a^2)}. \quad (10)$$

The wall (y=0, z=x) is insulating if the magnetic field is normal, that is, the tangential component or real part of $H^*$ is zero; that is the case for a $H_{+}^{*}$ equal image. Whereas if an equal image current corresponds to an insulating wall, the opposite image current (for $H_{-}^{*}$ ) leads to a tangential magnetic field that specifies the surface electric current distribution on the conducting wall:

$$\nu(x) = cH_x^{-}(x,0) = \frac{-Ja/\pi}{x^2+a^2} \quad (11)$$

The surface current is opposite to the original current, peak at the closest point, and leads to a total current $-J$, as in equations for the induced electric charge in a conductor. The surface electric current arises from boundary condition for a zero magnetic field on the other side of a perfect conductor. The complex conjugate magnetic force on the boundary is minus the force of the image on the line current, and is given by:

$$F_{\pm}^{*} = \frac{i\mu J}{c}\left[H_{+}^{*} - \frac{i\mu J}{2\pi c}\left(\frac{1}{z-ia}\right)\right]_{z=ia}$$

$$= \frac{i\mu J}{c}\left[\pm\frac{i\mu J}{2\pi c}\left(\frac{1}{z-ia}\right)\right]_{z=ia}$$

$$= \frac{i\mu J}{c}\left(\frac{\pm iJ}{2\pi c}\right)\frac{1}{2ia}$$

$$= \pm\frac{i\mu J^2}{4\pi c^2 a} = -F_{\pm} \quad (12)$$

Only the second term on the right hand side of equation (8) appears in equation (12) because: (i) the line current does not exert a force on itself; (ii) the force on the line-current is entirely due to its image. Thus, the line current exerts on the insulating or conducting wall a normal repulsive or attractive force. Generalizing, to magnetic multipole of moment $P_n$, the complex conjugate magnetic field due to an insulating $H_+$ $(or\ conducting\ H_-)$ wall is:

$$H_{\pm}^{*}(Z) = \frac{inP_n}{2\pi c}\left[(z-ia)^{-n-1} \pm (z+ia)^{-n-1}\right]; \quad (13)$$

This corresponds in the far-field to:

$$|Z|^{n+1} \gg a^{n+1}: H_{+}^{*}(Z) = inP_n Z^{-n-1}; \quad (14)$$

$$H_{-}^{*}(Z) = -n(n+1)\frac{P_n}{\pi c}.aZ^{-n-2}$$

$$= i(n+1)\frac{P_{n+1}}{2\pi c}.Z^{-n-2}; \quad (15)$$

$$P_{n+1} = i2naP_n \quad (16)$$

The induced electric charges are similar to the interface electric currents:

$$\upsilon(x) = cH_x^{-}(x,0) = \frac{inP_n}{2\pi}\left[(x-ia)^{-n-1} - (x+ia)^{-n-1}\right]$$

$$= \frac{-nP_n}{\pi}(x^2+a^2)^{-n-1}I_m\{(x+ia)^{n+1}\} \quad (17)$$

Where a real dipole moment is taken. Using binomial theorem, the complex case is:

$$\upsilon(x) = \frac{-nP_n}{\pi}(x^2+a^2)^{-n-1}I_m\left\{\sum_{k=0}^{n+1}\binom{n+1}{k}(ia)^k x^{n+1-k}\right\}$$

$$= \frac{-nP_n}{\pi}(x^2+a^2)^{-n-1}\sum_{p=0}^{n/2}\frac{(-1)^p(n+1)!a^{2p+1}x^{n-2p}}{(2p+1)!(n-2p)!} \quad (18)$$

Since only the odd powers are imaginary. Thus, the multipole of order n, at a distance "a" from a conducting plane corresponds to an interface electric current distribution; its value at the closest point, x=0, is zero for odd n and non-zero for even n=2m:

$$n = 2m:$$

$$\upsilon(0) = \frac{-nP_n}{\pi}a^{-2n-2}(-1)^m.a^{2m+1}$$

$$= (-1)^{m+1}\left(\frac{2m}{\pi}\right)P_m a^{-2m-1} \quad (19)$$

$$\nu(0) = CH^{*}_{-(0)=\frac{inP_n}{2\pi}\{(-ia)^{-n-1}-(-ia)^{-n-1}\}}$$

$$\frac{inP_n.a^{-n-1}}{2\pi}\left\{e^{i\pi(n+1/2)} - e^{-i\pi(n+1)/2}\right\}$$

$$= \frac{-nP_n}{\pi}.a^{-n-1}sin\left[\frac{(n+1)\pi}{2}\right]$$

$$= (-1)^{m+1}\left(2m/\pi\right)P_{2m}.a^{-2m-1} \quad (20)$$

## 3. Cylinder in an External Uniform Magnetic Field

The complex potential of a cylinder of radius "a" in a uniform external magnetic $H_o$ is:

$$F_m^{\pm}(z) = -H_o\left(z \pm \frac{a^2}{z}\right). \tag{21}$$

The corresponding conjugate magnetic field of the cylinder of radius "a" in the uniform external magnetic field $H_o$ is specified by:

$$H_{\pm}^{*}(z) = H_o\left(1 \pm \frac{a^2}{z^2}\right). \tag{22}$$

The corresponding scalar potential is given by:

$$\Phi_m^{\pm}(r,\phi) = -H_o\left(r \pm \frac{a^2}{r}\right).cos\phi \tag{23}$$

The field function is:

$$\psi_m^{\pm}(r,\phi) = -H_o\left(r \pm \frac{a^2}{r}\right).sin\phi \tag{24}$$

The polar components of the magnetic field are:

$$H_r^{\pm}(r,\phi) = H_o\left(1 \pm \frac{a^2}{r^2}\right).cos\phi; \tag{25}$$

And

$$H_\phi^{\pm}(r,\phi) = -H_o\left(1 \mp \frac{a^2}{r^2}\right).sin\phi \tag{26}$$

The above Equations (25, 26) show that:

i. The upper sign indicates that the cylinder is an equipotential and corresponds to an insulator, that is, zero tangential magnetic field:

$$\Phi_m^{+}(a,\phi) = 0 \tag{27}$$

$$= H_\phi^{+}(a,\phi), \tag{28}$$

$$\psi_m^{+}(a,\phi) = -2H_o a sin\phi, \tag{29}$$

$$\mathrm{H}_r^{+}(a,\phi) = -2H_o \cos\phi. \tag{30}$$

(ii) The lower sign indicates that the cylinder is a field line corresponding to a conductor, with the nonzero tangential magnetic field:

$$\psi_m^{-}(a,\phi) = 0 \tag{31}$$

$$= H_r^{-}(a,\phi), \tag{32}$$

$$\Phi_m^{-}(a,\phi) = -2H_o a cos\phi, \tag{33}$$

$$\mathrm{H}_\phi^{-}(a,\phi) = -2H_o sin\phi = \frac{v(\phi)}{c} \tag{34}$$

Equation (34) specifies the interface electric currents. Thus, an insulating (or conducting) cylinder of radius "a" in an external magnetic field vanishes along the axis, and peaks in the transverse direction with opposite signs; the total current is zero and its moment is horizontal and equal to:

$$\oint_0^{2\pi} v(\phi)\, a\, d\emptyset = -2H_{\circ}ca \oint_0^{2\pi} sin\phi\, d\phi = 0, \tag{35}$$

$$\oint_0^{2\pi} xv(\phi)ad\phi = -2\mathrm{H}_{\circ}ca^2 \oint_0^{2\pi} sin\emptyset cos\emptyset d\emptyset = 0, \tag{36}$$

$$\oint_0^{2\pi} yv(\phi)ad\phi = -2ca^2 \oint_0^{2\pi} sin^2\phi d\phi = -2\pi c\mathrm{H}_{\circ}a^2 \equiv \mathrm{P}_1 \tag{37}$$

The dipole moment in Eq. (37):

$$\mathrm{H}\_^*(Z)-\mathrm{H}_{\circ}=-\mathrm{H}_{\circ}a^2/ Z^2=\mathrm{P}_1/ (2\pi cZ^2) \tag{38}$$

Where $\mathrm{P}_1 \equiv -2\mathrm{H}_{\circ}a^2c$.

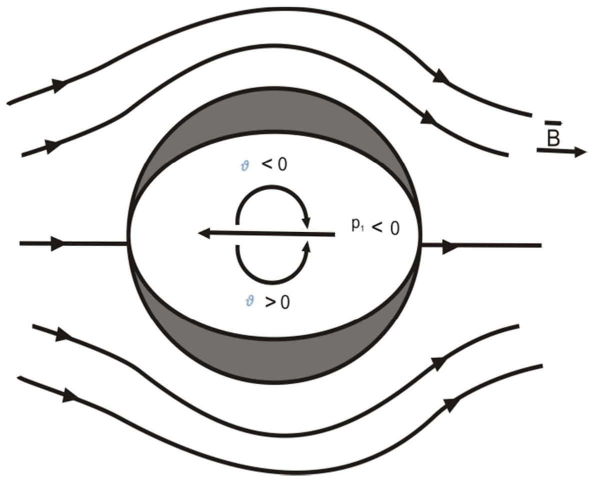


***Figure 1.*** *Conducting cylinder in a uniform magnetic ( or electric) field.*

The introduction of a conducting cylinder in a uniform magnetic (electric) field (Figure 1) implies a distribution of surface electric currents (or charges) such that the total magnetic (or electric) field is everywhere tangent (or orthogonal) to the cylinder, respectively. The positive and negative electric currents (or charges) concentrate at opposite ends of the cylinder in the direction through the axis orthogonal to (or along) the external magnetic (or electric) field. The total electric current (or charge) is zero and there is dipole moment along the external magnetic (or electric) field, respectively. The introduction of a conducting (or insulating) cylinder in an external magnetic or electric field is equivalent in both cases to a dipole parallel (or antiparallel) to the external field, respectively. Besides the preceding analogies and differences between the electro-(or magneto-) static field due to electric charges (or currents), in the case of a conductor: (i) a nearby electric charge induces a distribution of surface electric charges and the dielectric displacement is zero in interior; (ii) a nearby electric current does not induce a distribution of surface electric currents, but such a fictitious electric current distribution is needed to match a nonzero (or zero) tangential magnetic field outside (or inside**),** respectively. A conducting cylinder in a uniform external electric (or magnetic) field (as shown in Fig. 1) has induced electric charges (or surface electric currents) along (or across) the field, leading to a horizontal dipole moment, along the field in both cases. In both cases, there is no force and no torque on the cylinder because:

*The electric (or magnetic) force on the co-located, identical, and opposite electric charges (or current) cancel.

*The dipole axis is parallel to the uniform external field.

In the latter case of a conductor, it is possible to add a line-current along the axis of the cylinder:

$$f(z) = f_{-}(z) - \frac{iJ}{2\pi c} log z$$

$$= -H_o\left(z - \frac{a^2}{z}\right) - i\left(\frac{J}{2\pi c}\right) log Z \quad (39)$$

$$H*(\mathbb{Z}) = H^{*-}(\mathbb{Z}) - \frac{ij}{2\pi c\mathbb{Z}} = H_{\circ}\left(1 - \frac{a^2}{\mathbb{Z}^2}\right) = \left(\frac{ij}{2\pi c\mathbb{Z}}\right) \quad (40)$$

The above preserves the boundary condition {Eq. (32) ≡ Eq.(41)}:

$$H_{\mathrm{r}}(a, \emptyset) = 0, \quad (41)$$

$$v(\phi) = cH\emptyset(a, \emptyset) = \frac{-j}{2\pi a} - 2H_{\circ}\sin\phi; \quad (42)$$

which adds a constant term to the current (42) relative to (34).

# 4. Line – Current Near an Insulating or Conducting Cylinder

A cylinder of radius "a" placed near a line-current J at a distance b corresponds to the complex potential (43) [or conjugate magnetic field (44)]; it is specified by the circle theorem, viz.:

$$F_{m}^{\pm}\mathbb{Z} = \frac{ij}{2\pi c}\left[log(\mathbb{Z} - b) \pm log\left(\frac{a^2}{\mathbb{Z}} - b\right)\right] \quad (43)$$

$$H_{+}^{*}(Z) = \frac{iJ}{2\pi c}\left[\frac{1}{\mathbb{Z}-b} \pm \frac{a^2}{\mathbb{Z}^2 b - a^2\mathbb{Z}}\right] \quad (44)$$

On the cylinder, the arguments of the logarithms in equation (43) have the same modulus (or opposite phases), and thus the same modulus in square brackets is imaginary (or real) for the lower (or upper) sign respectively and thus $f_{-}$ (or $f_{+}$) is real (or imaginary). Hence, the cylinder is a field-line $\psi$=0 (or equipotential $\Phi$=0), corresponding to a conductor (or insulator).

Thus, a line-current J at a distance b from the center of an insulating (or conducting) cylinder of radius "a" has complex magneto-static potential (43) and complex conjugate magnetic field (44) with upper (or lower) sign, respectively.

In the conducting case, the corresponding surface electric currents are:

$$v(\phi) = cH_{\varphi}^{-}(a, \phi) = \left({}^{j}/_{\pi}\right)\frac{[bcos\phi - a]}{[a^2 + b^2 - 2abcos\phi]} \quad (45)$$

These are largest (or smallest) in modulus and closest $\phi = 0$ (or farthest$\phi = \pi$) from the line-current.

In both the conductor and insulator cases, the far-field is due to the leading order to the electric current alone:

$$|\mathbb{Z}|^2 >> b^2 : H_{\pm}^{*}(\mathbb{Z}) = \frac{ij}{(2\pi c\mathbb{Z})}\left(1 + \frac{b^2 \pm \frac{a^2}{b}}{\mathbb{Z}}\right) \quad (46)$$

Because there are two opposite images on the cylinder, we have:

$$log\left(\frac{a^2}{\mathbb{Z}} - b\right) = log\left[\frac{-b}{\mathbb{Z}}\left(\mathbb{Z} - \frac{a^2}{b}\right)\right] = \log\left(\mathbb{Z} - \frac{a^2}{b}\right) - \log\mathbb{Z} + \log(-b) \quad (47)$$

These appear as a dipole moment (Eq. 48):

$$P^1 = J\left(b \pm \frac{a^2}{b}\right) \quad (48)$$

$$H_{\pm}^{*}(\mathbb{Z}) = \frac{iJ}{2\pi c\mathbb{Z}} + \frac{iP_1}{2\pi c\mathbb{Z}^2} + O\left(\frac{b^2}{\mathbb{Z}^3}\right) \quad (49)$$

The second term of Eq. (46) is equivalent to Eq. (49). The dipole moment ($\mathrm{P}_1$) is horizontal, and for the conductor (or insulator) is the distance from the external current to the reciprocal point on the same (or opposite) side of the origin, respectively.

The complex conjugate magnetic force exerted on the cylinder by the line-current is minus that due to its image, that is, the second term on the right-hand side of Eq. (44) evaluated at $\mathbb{Z} = b$:

$$F_{\pm}^{*} = \frac{i\mu J}{c}\,\frac{(\pm iJa^2)}{2\pi cb(b^2 - a^2)} = \mp\frac{\mu J^2}{c}\,\frac{\left(\frac{a}{b^2}\right)}{\left[b - \frac{a^2}{b}\right]} = F_{\pm} \quad (50)$$

Eq. (50) implies that there is attraction (or repulsion) on a conducting (or insulating) cylinder, that is, lower+(or upper-) sign in Eq. (50). The force changes sign if the line-current is inside the cylinder, so that it is always a force of attraction (or repulsion) for a conductor (or insulator), respectively.

The force (Eq. (50)) involves:

1. The inverse of the distance from the line-current at b to its image at $a^2/b$.

2. There is a factor $(a/b)^2$ smaller for line charge and farther by b from the centre of the cylinder of radius a.

The expression for the force (Eq. (50)), as well as those for complex potential (Eq. (43)), conjugate magnetic field (Eq. 44), their asymptotic forms (Equations (46), (48, 49)), and surface electric current (Eq. (45)), apply to a line-current at distance, b, from the center of the cylinder of radius a, regardless of whether it lies inside b < a or outside b > a.

# 5. Real, Image and Virtual Electric Currents

An electric line-current J at a distance b from the centre of a cylindrical interface of radius "a" between media of magnetic permeability $\mu_1$ $(or\ \mu_2)$ for r > a (or r < a) leads to a complex conjugate magnetic field specified by:

(i) In the medium (r > a) of the line-current J by superposition with the image current $J^{II}$ on the circle, we have:

$$r < a : H_1 * (\mathbb{Z}) = \frac{i}{2\pi c}\left[\frac{J}{\mathbb{Z}-b} + \frac{J^1 a^2}{\mathbb{Z}(b\mathbb{Z} - a^2)}\right] \quad (51)$$

$$r > a : H*^2(\mathbb{Z}) = \frac{i}{2\pi c}\frac{J^{II}}{(\mathbb{Z}-b)} \quad (52)$$

(ii) In the medium (r > a) without line-current, J, the magnetic field is due to a virtual current $J^{II}$ at the original point. The boundary conditions at the interface without surface electric current:

$$\nu = 0: H1_{\varphi}(a, \phi) = H_{2\varphi}\ (a, \phi) \tag{53}$$

$$\mu 1\ H1r(a, \phi) = \mu_2 H_2 r(a, \phi) \tag{54}$$

are used to relate the original current J, image current $J^{I}$ and virtual current $J^{II}$.

The components of the magnetic field on the cylindrical interface are:

$$Hr^1, Hr^2(a, \phi) = \frac{1}{2\pi c}\left\{\frac{J}{R} + \frac{J^{I}a}{bR^{I}}, \frac{J^{II}}{R}\right\}\sin\psi \tag{55}$$

$$H_{\varphi i}, H_{\varphi 2}\ \ (a, \phi) = \frac{1}{2\pi c}\left\{\frac{J}{R} - \frac{J^{I}a}{bR^{I}}, \frac{J^{II}}{R}\right\}\cos\psi \tag{56}$$

Where:

R=distance from an arbitrary point on the circle to the original current at (b, 0);

$R^{I}$=distance from an arbitrary point on the circle to the reciprocal point at ($a^2$/b, 0); and

$\psi = angle\ with\ the\ real\ axis.$

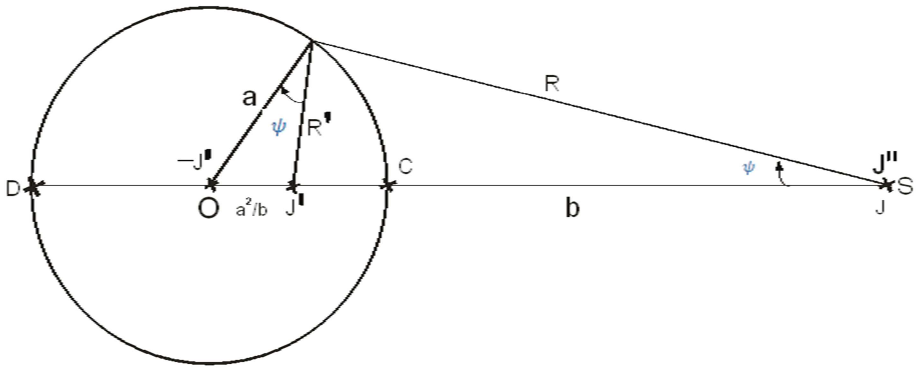


***Figure 2.*** *Analogy between magnetic field of a line-current.*

There is an analogy between the magnetic (or electric) field of a line current (or charge) near a cylindrical (or plane) interface between media with distinct magnetic permeabilities (or dielectric permittivities), respectively. The image system consists of current $J^{II}(\pm J^{I})$ at the original point (or reciprocal point and center of the cylinder). The relation between the original J and image $J^{I}$, $J^{II}$ currents is specified by the continuity of the magnetic field components orthogonal (or tangent) to the cylinder, which holds always (in the absence of surface electric currents). The extreme cases of a conducting (or insulating) cylinder correspond to zero $\mu_2 = 0$ (or infinite $\mu_2 = \infty$) magnetic permeability of the second medium and opposite (or identical) image current $J^{I} = -J(J^{I} = J)$, respectively. There is a continuous variation between these extremes, allowing an attraction $\mu_2 < \mu_1$ (or repulsion $\mu_2 > \mu_1$) of the cylinder by the line current, and no force for identical media.

The physical configuration is shown in Fig. 2. Substituting (55-56) in the boundary conditions (53), we get:

$$\frac{J}{R} - \frac{J^{I}a}{bR^{I}} = \frac{J^{II}}{R}, \tag{57}$$

$$\mu\left(\frac{J}{R} + \frac{J^{I}a}{bR^{I}}\right) = \mu_2\left(\frac{J^{II}}{R}\right) \tag{58}$$

The magnetic field $\vec{H}$ $(or(\vec{H})^{II})$ due to the original J (or virtual$J^{II}$) line-current (Fig. 3) is orthogonal to the line from the original current J to the arbitrary point P on the circle, in the positive direction. The magnetic field $H^{II}$ due to the image line-current $J^{I}$ has components due to a line-current $J^{I}$ (or-$J^{I}$) at the reciprocal point (or at the center). It can be seen that $\overrightarrow{(H)'}$ has a radial (or tangential) component with the same (or opposite) sign to $\vec{H}$ and $\overrightarrow{(H^{II})}$.

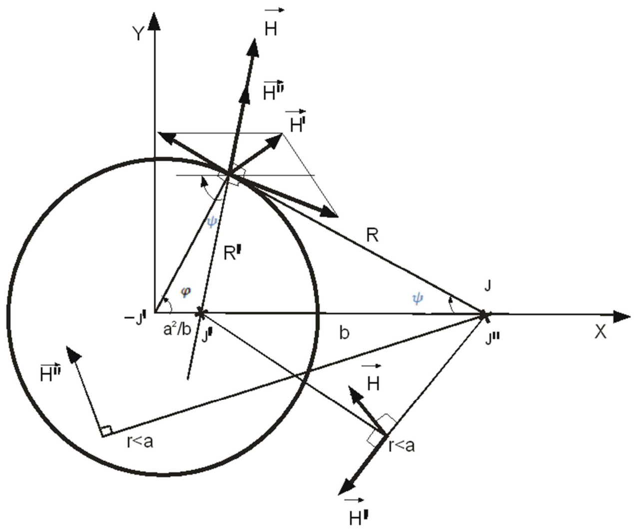


***Figure 3.*** *Image at the reciprocal point in a circle.*

The image at the reciprocal point in a circle applies to: (i) the electric (magnetic) field of a line charge (current) near a cylinder (Figure 2); (ii) the potential flow of a line monopole (dipole) near a cylinder. The original (reciprocal) point have the property that the ratio of the distance from them at any point on the circle $\frac{R^{I}}{R} = \frac{b}{a}$ equals the distance of the original point from the center of the circle divided by the radius of the circle. this can be checked most readily for 2 points on the circle farthest D (closest C) to the reciprocal points $\frac{(a \pm a^2/b)}{(b \pm a)} = a/b$. The magnetic fields $\vec{H}\left(\vec{H}^{II}\right)$ due to the original J (virtual $J^{II}$) line-current are orthogonal to the position vector from (b, 0) to the circle; the magnetic field $H^{I}$

is the sum of the contributions due to the opposite image line-currents $J^I(-J^I)$ *at the reciprocal* point $(a^2/b, 0)$[center (0, 0)].

# 6. Reciprocal Point as Image on the Cylinder

In Equations (57, 58), the distance from an arbitrary point (P in Figure 3) on the circle (a, $\phi$) to the original and virtual (or image) currents J, $J^{II}$ (or $J^I$) at (b, 0) [or at the reciprocal point ($a^2$/b, 0)] is denoted by R (or $R^I$) in Eq. (59) [or Eq. (60)]:

$$R^2 \equiv \left|ae^{i\emptyset} - \frac{a^2}{b}\right|^2 = (b - a\cos\emptyset)^2 = (a\sin\emptyset)^2 = a^2 + b^2 - 2ba\cos\emptyset \quad (59)$$

$$(R^{\text{I}})^2 \equiv \left|ae^{\text{i}\phi} - \frac{a^2}{b}\right|^2 = \left(a\cos\phi - \frac{a^2}{b}\right)^2 + (a\sin\phi)^2 + a\sin\phi)^2 = a^2 + \frac{a^4}{b^2} - 2\frac{a^3}{b}\cos\phi \quad (60)$$

These are related by (Eq. (61)):

$$a^2R^2 = a^4 + b^2a^2 - 2ba^3\cos\phi = b^2(R^{\text{I}})^2; \quad (61)$$

$$\frac{R^{\text{I}}}{R} = a/b, \quad (62)$$

that is equivalent to Eq. (62). This leads to the reciprocal point theorem (Equation (63)) illustrated in Fig. 2:

The distances from an arbitrary point on a circle of radius “a” to an external point (b, 0) [the reciprocal point being ($a^2$/b, 0)] are in a constant ratio, equal to a/b. This can be checked in particular for the points on the circle, that are closest (or farthest) from source:

$$R = b \mp a; \quad (63)$$

$$R^I = a \mp a^2/b \quad (64)$$

$$R^I/R = a/b \quad (65)$$

These correspond to C (or D) in Figure 2, with upper (or lower) sign, both leading to the same result.

# 7. Homogeneous Medium and Conducting or Insulating Walls

Substituting the reciprocal point theorem (Eq. (62)) in [Equations (60, 61)], we obtain:

$$J - J^I = J^{II} \quad (66)$$

$$J + J^I = (\mu_2/\mu_1)J^{II} \quad (67)$$

These specify the image $J^I$ and virtual $J^{II}$ currents in terms of the original current J, viz.:

$$J^I = J\,[(\mu_2 - \mu_1)/(\mu_2 + \mu_1)] \quad (68)$$

$$J^{II} = J[(\mu_1)/(\mu_2 + \mu_1)] \quad (69)$$

where $\mu_1$ and $\mu_2$ are the magnetic permeabilities of the media on the two sides of the magnetic cylindrical interface.

A line-current J has conjugate magnetic field:

$$F_{\text{M}}(Z) = \frac{-iJ}{2\pi c}\log(Z - b), \quad (70)$$

$$H*(Z) = \frac{-dF_{\text{M}}}{dZ} = \frac{iJ}{2\pi c}\left(\frac{1}{Z-\text{s}}\right), \quad (71)$$

where $f_m$ (z)=complex potential;

$H^*(z) = conjugate\ magnetic\ field.$

Also, we have:

$$f_m(z) = \frac{-i}{2\pi c}\int j(\varsigma)\log(z-\varsigma)d\varsigma \quad (72)$$

$$H*(Z) = \frac{i}{2\pi c}\int j(\varsigma)\,(z-\varsigma)^{-1}d\varsigma \quad (73)$$

As for the plane interface between two dielectrics, two particular cases are considered beside the degenerate case of identical magnetic permeabilities. In the degenerate case of identical media:

$$\mu_1 = \mu_2; \quad (74)$$

$$J^I = 0; \quad (75)$$

$$J^{II} = J, \quad (76)$$

when there is no image current or interface, and the virtual current coincides with the real current (Equation (76)), so that the magnetic field:

$$H_1^*(Z) \equiv H_2^*(Z) \equiv H*(Z)$$

Which is that of a line-current in free space.

One extreme case is the interior of a cylinder with zero magnetic permeability, (equation (77)), implying zero magnetic induction (equation (78)), though the magnetic field may be non-zero:

$$\mu_2 = 0; \quad (77)$$

$$B_2(z) = 0 \neq H_2(z); \quad (78)$$

$$J^I = -J; \quad (79)$$

$$J^{II} = 2J. \quad (80)$$

The image current is opposite in direction to the real or original current, corresponding to a conducting cylinder (Equation (52)); the virtual current is double the original current (Equation (80)), in agreement with the non-zero magnetic field.

The opposite extreme case is when the magnetic permeability of the interior of the cylinder is infinite (Eq. (81)), so that the magnetic field is zero (Eq. (82)), though the magnetic induction may not be zero:

$$\mu_2 = \infty; \quad (81)$$

$$H2(z) = 0 \neq B2(z); \quad (82)$$

$$J^I = J; \quad (83)$$

$$J^{II} = 0. \tag{84}$$

The image current is identical to the real or original current, corresponding to an insulating cylinder (Eq. (52)). The virtual current vanishes, in agreement with the nonexistence of magnetic field there.

# 8. Magnetic Force and Effective Dipole Moment in the Far-Field

A line – current J at a distance b from the center of a cylindrical interface of radius "a" between two media with magnetic permeabilities, $\mu_1, \mu_2$, creates a magnetic field inside or outside corresponding to a virtual current or an image current.

The far-field (Eq. (85)) in the first medium outside the cylinder corresponds to the original line-charge plus a dipole (Eq. (86)):

$$|z|^2 \gg b^2 \tag{85}$$

$$H_1^* \sim \frac{iJ}{2\pi cz} + \frac{iP_i}{2\pi cz^2} \tag{86}$$

$$P_1 \equiv Jb\left[\frac{\mu_1(1-a^2/b^2)+\mu_2(1+a^2/b^2)}{(\mu_1+\mu_2)}\right] \tag{87}$$

Which has an effective dipole moment (87) that simplifies to (J. b) for line-charge at a large distance from the cylinder, where $a^2 \ll b^2$.

The result follows from (Equations (51, 52)):

$$\frac{2\pi c}{iJ} H_I^*(Z) - \frac{1}{Z} + \frac{b}{Z^2} + 0(b^2/Z^3) = \left(\frac{J^I}{J}\right)\left[\frac{[a^2/(bz^2)]}{[1 - a^2/(bz)]}\right]$$

$$= \frac{\mu_2 - \mu_1}{\mu_2+\mu_1}\left[\frac{a^2}{bz^2} + 0\left(\frac{a^2}{bz^2}\right) + 0\left(\frac{a^4}{b^2z^3}\right)\right]; \tag{88}$$

The above equation simplifies to (86) with dipole moment:

$$P_1 = Jb\left[1 + (a^2/b^2)\left(\frac{\mu_2 - \mu_1}{\mu_2+\mu_1}\right)\right]; \tag{89}$$

$$0\left(b^2/z^3\right) \sim 0\left(b^2/z^3\right) 0\left(a^4/b^4\right) \tag{90}$$

Since the current lies outside the cylinder, b > a, the term (90) can be neglected together with $0\left(b^2/z^3\right)$, so that the asymptotic condition (85) is sufficient to omit both. The complex conjugate magnetic force exerted by the line-current on the cylindrical magnetic interface is minus the force due to its images, and is specified by the second term on the right-hand side (RHS) of Eq. (51) evaluated at the line-current $Ƶ = b, viz.$:

$$\tilde{F}^* = \left(\frac{i\mu^1 J}{c}\right)\left(\frac{iJ^{\mathrm{I}}}{2\pi c}\right)\left(\frac{a^2/b}{b^2-a^2}\right) = -\left(\frac{\mu_1 J^2}{2\pi c^2}\right)\left(\frac{a^2/b^2}{b-a^2/b}\right)\frac{(\mu_2-\mu_1)}{(\mu_2+\mu_1)} = \tilde{F}_{\mathrm{M}} \tag{91}$$

Hence, the force exerted by the line-current on the cylindrical interface is attractive (or repulsive) if the force medium has larger (or smaller) magnetic permeability than the second medium, respectively.

It vanishes for identical media, and in the particular case $\mu = \infty$ (or $\mu = 0$), it reduces to (50) with upper – (or lower+) sign, corresponding to an insulator (or conductor), respectively.

# 9. Analogies and Differences Between Electrostatic and Magnetostatic Fields

***Table 1.*** *Comparison of Electro-and Magnetostatics.*

| Statics | Electro-case | Magneto-case |
|---|---|---|
| In vacuo | Electric field | Magnetic induction |
| Equation | $\nabla \wedge \vec{E}$ | $\nabla . \vec{B} = 0$ |
| Type | Irrotational | Solenoidal |
| Representation by | $\vec{E} = -\nabla\Phi$ | $\vec{B} = \nabla\Lambda\Psi$ |
| Existence of | Scalar potential | Field function (2-D) |
| Electric | Charge density: q | Current density: $\vec{\jmath}$ |
| Field | Electric displacement | Magnetic field |
| Equation | $\nabla . \vec{D} = q$ | $\nabla \Lambda \vec{H} = \vec{\jmath}/c$ |
| Boundary | Charge: $\sigma$ | Current: $\vartheta$ |
| Normal | $[D_n] = \sigma$ | $[B_n] = 0$ |
| Tangential | $[E_t] = 0$ | $[H_t] = \vartheta/c$ |
| Constitutive parameter | Dielectric permittivity: $\epsilon$ | Magnetic permeability: $\mu$ |
| Constitutive equation | $\vec{D} = e\vec{E}$ | $\vec{B} = \mu\vec{H}$ |
| Poisson equation | $\nabla^2\Phi = -q/\in$ | $\nabla^2\Psi = -\mu j/c$ |
| Insulator | $\in = 0$ | $\mu = \infty$ |
| Charge/current | $\sigma = 0$ | $\vartheta = 0$ |
| Field | $\vec{E} \neq 0$ | $\vec{H} = 0$ |
| Displacement/Induction | $\vec{D} = 0$ | $\vec{B} \neq 0$ |
| Conductor | $\in = \infty$ | $\mu = 0$ |
| Charge/current | $\sigma \neq 0$ | $\vartheta \neq 0$ |
| Field | $\vec{E} = 0$ | $\vec{B} = 0$ |
| Displacement/Induct-ion | $\vec{D} \neq 0$ | $\vec{H} \neq 0$ |

Note: The comparison of electro-and magneto-statics concerns: (i) the electric field ( or magnetic induction) vectors and potential (or field) function: (ii) the electric charge (or current) and electric displacement (or magnetic field) vectors; (iii) the dielectric permittivity (or magnetic permeability) as the constitutive parameter in the Poisson equation; (iv) the extreme cases of an insulator; (v) the extreme cases of a conductor.

For a summary of this section, refer to Table 1.

The image current $J^I$ and virtual current $J^{II}$ can be rewritten respectively as:

$$J^{\mathrm{I}} = \frac{J}{\left(\frac{1}{\mu_1}+\frac{1}{\mu_2}\right)}\left\{\frac{1}{\mu_1} - \frac{1}{\mu_2}\right\} \tag{92}$$

$$J^{\mathrm{II}} = \frac{J}{\left(\frac{1}{\mu_1}+\frac{1}{\mu_2}\right)}\left\{\frac{1}{\mu_1} - \frac{1}{\mu_2}\right\} \tag{93}$$

The equations above are specified by the same equations as the image (or virtual) electric charges.

We have:

$$\frac{1}{\mu} \Leftrightarrow \mathcal{E}; \tag{94}$$

$$\Delta^2\phi_{\mathrm{e}} = \frac{-q}{\mathcal{E}.}, \tag{95}$$

$$\Delta^2\Psi_{\mathrm{M}} = -\mu j/c \tag{96}$$

In (94), we are exchanging the dielectric permittivity by the inverse of the magnetic permeability. Poisson equations for the electric potential and for the magnetic field function are given by Equations (95) and (96), respectively. The corresponding electrostatic and magneto-static analogies in the Poisson equations are:

$$\phi_e \leftrightarrow \Psi_M; \tag{97}$$

$$q \leftrightarrow {}^{j}/_{c}; \tag{98}$$

$$\frac{1}{\varepsilon} \leftrightarrow \mu; \tag{99}$$

$$\frac{1}{\mu} \leftrightarrow \varepsilon. \tag{100}$$

The virtual and image electric charges (or currents) are the same for the plane or cylindrical interfaces between distinct media; for both types of interface the dielectric permittivity (or magnetic permeability) is placed by the inverse magnetic permeability (or inverse dielectric permittivity) when exchanging magnetic for electric fields.

By applying the required transformations, it follows that the magnetic (or electric) force exerted by a line-current (or charge) at a distance “a” from a plane interface (or distance b from some cylindrical interface with radius “a”), between media with magnetic permeabilities,

$\mu_1$, $\mu_2$ (*or with dielectric permittiv*ities $\varepsilon_1$, $\varepsilon_2$), is given by:

$$F_M^* = \frac{-i({}^{j}/_{c})^2}{2\pi a/_{\mu_1}}\left[\frac{{}^1/_{\mu_1}-{}^1/_{\mu_2}}{{}^1/_{\mu_1}+{}^1/_{\mu_2}}\right] = i\left(\frac{\mu_1 J^2}{2\pi c^2 a}\right)\left[\frac{\mu_2-\mu_1}{\mu_2+\mu_1}\right] \tag{101}$$

$$\tilde{F}_M^* = \frac{-e^2/_{\varepsilon_1}}{2\pi}\left(\frac{({}^{a}/_{b^2})}{b-{}^{a^2}/_{b}}\right)\left[\frac{{}^1/_{\varepsilon_2}-{}^1/_{\varepsilon_2}}{{}^1/_{\varepsilon_2}+{}^1/_{\varepsilon_1}}\right] = \frac{-e^2}{2\pi\varepsilon_1}\left(\frac{({}^{a}/_{b})^2}{b-{}^{a^2}/_{b}}\right)\left[\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\right]. \tag{102}$$

Comparing the plane (or cylindrical) interface both in the electrostatic and magnetostatic cases, the curvature effect is the same in ratio between a line-monopole ( or charge) at a distance: (i) b from a plane interface; (ii) b from the center of a cylindrical interface of a radius “a”.

That is, we have:

$$\frac{i\tilde{F}_e^*}{F_e^*} = \frac{-i\tilde{F}_e}{F_e} = \left|\frac{\tilde{F}_e}{F_e}\right| = \left|\frac{\tilde{F}_m}{F_m}\right|$$

$$= \frac{-i\tilde{F}_m}{F_m} = \frac{-i\tilde{F}_m^*}{F_m}$$

$$= \frac{(a/b)^2}{(b-a^2/b)} : \frac{1}{2b}$$

$$= \frac{2a^2}{(b^2-a^2)} = \frac{2}{[b^2/a^2-1]} = K. \tag{103}$$

Hence, it can be seen that:

(1) The curvature effect (of the cylindrical interface) increases the force, for K > 1 and for close monopole a < b < a.$(3)^{1/2}$;

(2) The curvature effect cancels for K=1 and for a monopole at a distance b=a. $(3)^{1/2}$;

(3) The curvature effect decreases the force, for b > a.$(3)^{1/2}$ causing a decay ~ $2a^2/b^2$ at large distance (s) $b^2 >> a^2$.

An insulator does not support electric charges (or currents), so e=0 (or J=0), and the scalar potential (or stream function) in the Poisson equation can be finite and nonzero only if $\in = 0$ $(and\ \mu = \infty)$ which corresponds to zero electric displacement (or zero magnetic field), though the electric field (or the magnetic induction) may be nonzero. The reverse happens for a conductor that supports electric charges (or currents) so that for a perfect conductor $\in = \infty$ (and $\mu = 0$), and the electric field $\vec{E}$ (and magnetic induction $\vec{B}$ ) are zero, though the electric displacement D (and magnetic field $\vec{H}$) may be nonzero.

Thus, the Faraday cage is formed in a conductor which conducts and excludes not only the electric field but also the magnetic induction. This leads to the phenomena of “skin effect”. In this phenomenon, the current along the central axis of a conductor (or metal wire) is small, while a large part of the current is concentrated along the periphery or boundaries of the conductor. At very high frequencies, these (power or information) currents or signals are emitted into space. Hence, this phenomenon makes possible the designs of antennas.

# 10. Conclusions

In conclusion, it can be correctly asserted that this paper has treated the following cases: current near conducting or insulating plane, cylinder in a uniform external magnetic field, line current near an insulating or conducting real, image and virtual electric currents, and the consideration of a reciprocal point as image on the cylinder. The reciprocal point theorem was stated and proved for a reciprocal point on the cylinder being considered.

One extreme case is that the interior of the cylinder with zero magnetic permeability, implying zero magnetic induction, although the magnetic field may be nonzero. For this case, the image current is opposite to the real current, corresponding to a conducting cylinder, and the virtual current is double the real current.

The opposite extreme case is when the magnetic permeability of the interior of the cylinder is infinite, so that the magnetic field is zero, though the magnetic induction may not be zero. In this case, the image and real currents are identical corresponding to an insulating cylinder. Also observed is the fact that a line-current J at a distance b from the center of a cylindrical interface of radius “a” between two media with magnetic permeabilities, $\mu_1$, $\mu_2$, creates a magnetic field outside and inside corresponding to image and virtual currents respectively; the far-field in the first medium outside the cylinder corresponds to the original line-charge plus a dipole. The force exerted by the line-current on the cylindrical interface is attractive (or repulsive) if the first medium has larger (or smaller) magnetic permeability than the second medium. The exerted force vanishes for identical media, and in the particular case of $\mu_2 = \infty$ $(or\ \mu_2 = 0\,)$, it reduces to Equation (50) with upper-(or lower+) sign, corresponding to an insulator (or conductor) repectively. Expressions have been obtained for a magnetic and electric forces exerted by a line-current (or charge) at a distance "b" from a cylindrical interface with radius "a", between media with magnetic permeabilities, $\mu_1, \mu_2$, and with dielectric permittivities $(\in_1, \in_2)$.

Comparing the plane interface with cylindrical interface, both in the electrostatic and magnetostatic cases, the curvature effect is the same in ratio between a line-monopole at a distance "b" from the centre of a cylindrical interface of radius "a".

The curvature effect due to the cylindrical interface increases, cancels, or decreases the values of the magnetic and electric forces, depending on the values of ratio K and of distance "b" with respect to the radius "a" of the cylindrical interface. The skin effect is mentioned, which is applied in antenna design principles.

## References

[1] Coleman, S., 1995, Aspects of symmetry (Pergamon, Cambridge).

[2] Foerster, D. 1999, Physics letters 87, 87-93.

[3] Foerster, D., H. B. Nielsen and M. Nimomiya, 2003, Phys. Letters, B-94, 135-139.

[4] Read, N and D. Green, 2005, Physics Rev., B-96, 331-335.

[5] Wen, X. G. and Q. Niu, 2007, Phys. Rev., B-98, 560-567.

[6] Walker, Jearl (2007): "The Fundamentals of Physics", Wiley & Co., 8th Ed., pp. 567-901.

[7] Bateman, H. (2008): "Electrical And Optical Wave Motion", Cambridge University, 8th Ed., pp. 113-121.

[8] Slater, L. J. and Frank, N. (2009): "Electromagnetism", McGraw-Hill, 10th Ed., pp. 89-95.

[9] Ceninkcent, S. (2011): "Mechatronics", Wiley & Co., 2nd Edition, pp. 67-73.

[10] Http://www.current-magnetic-interactions.com/cylindrical-interfaces (July, 2016).